AMYGDALA

$$z \mapsto z^2 + c$$

FIRST WORDS

Thanks to the kindness of A.K. Dewdney, who took a chance and put a notice about this newsletter in his Computer Recreations column (**Scientific American**, August 1986) without seeing a copy of it, **AMYGDALA** comes into existence. Its purpose is to promote communication and interchange of ideas about the mathematical, computational, graphical, aesthetic, etc. aspects of the Mandelbrot Set (hereafter called "$\mathcal{M}$") based on articles, pictures, and notes sent in by you, its readers, as well as by me (Rollo Silver).

We'll also have announcements of freeware, shareware, and $$ware programs that contributors have to offer. We **won't** be a dealer for these, however.

Some of you are programmers, some not; some have the math, some not. That means that material in the newsletter is going to be over the heads of some, beneath notice for others, just right for still others. However, I hope you will all enjoy it, especially the pictures!

It's OK to copy this publication and give it out (but not to sell it!).

CONTRIBUTIONS AND PUBLICATION

This newsletter was composed on an Apple Macintosh using MacWrite, MacPaint, and ReadySetGo, printed on an Apple LaserWriter, and xeroxed. Future issues will be offset printed.

For those of you that have Macs and want to contribute pictures, text, programs, or whatever, please send me the material on single sided, single density diskette (either MFS or HFS format) if you can. If you want the diskette returned to you, please enclose a SASE (39¢).

Other material is welcome, just less convenient. Sorry that I don't have an IBM AT, Cray, color Xerox, etc. I assume that anything sent to me is submitted for publication unless otherwise specified.

Color pictures should be sent in the form of slides.

I suggest that all $\mathcal{M}$-pix have locating information associated with them, including center coordinates, magnification, pixelation (#pixels across & down), as well as dwell limit, precision, and coloring scheme (if you know them). Computer used (with special equipment noted), program used, and time required to generate would also be nice.

COLOR PHOTOS

I'm working on generating color photos having 1152x900 pixels using a palette of 256 colors out of 16,777,216. One or

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more of these will be sent out with issue #1 or #2 of Amygdala.

NOTATION

There are lots of ways to describe the frame (rectangular window) in which a detail of $\mathcal{M}$ is displayed: coordinates of lower left / upper right corners, lower left and dimensions, etc. I like to use a notation based on three numbers: (1) the complex number representing the point at the center of the rectangle, (2) the "magnification" 1/R, where R is the radius of the largest circle that will fit inside the rectangle, and (3) the pixelation XxY.

For example, my program, described below under **SPEEDUP**, produced the usual black & white picture of $\mathcal{M}$ in the rectangle from -2-i to 0.5+i, which has frame -.75(1x; 251x301); i.e. the center is at -.75+0i, the magnification is 1, and the grain size is 251x301 pixels.

The center should be given to a precision sufficient to specify it to within 1%(say) of the radius of the picture; e.g. the two decimal places above.
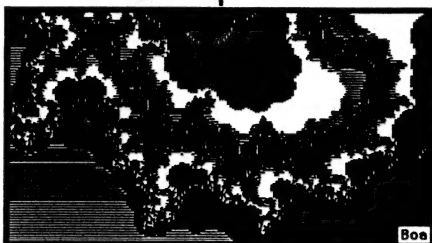
TERMINOLOGY

The **iterates** of a complex number c are the numbers $z_0 = 0$, $z_1 = z_0^2 + c$, $z_2 = z_1^2 + c$, ...

I call the smallest **n** for which $|z_n| > 2$ the **dwell** of c. If all $|z_n| \le 2$, i.e. $c \in \mathcal{M}$, then the dwell is infinite.

SPEEDUP

I programmed the Macintosh to display $\mathcal{M}$ in black and white, in the frame -.75 (1x; 251x201), using a dwell cutoff of 1024,

i.e. if $|z_n| \leq 2$ for $n \leq 1024$, I assume $c \epsilon \mathcal{M}$. It took only 1095 seconds to compute and display the 50,451 points, of which 15,048 were in $\mathcal{M}$. The remaining 35,353 points had an average dwell of 8.4. The program is mostly written in C, but the heart of it -- the calculation of dwell -- is written in Assembler using fixed-point arithmetic (32 bits).

Three tricks are used in the dwell calculation to detect $c \epsilon \mathcal{M}$ without having to do 1024 iterations. The slow/fast trick, described in Dewdney's column (essentially: quit if $z_k = z_{k/2}$) accounted for 9358 (62%) of the $c \epsilon \mathcal{M}$. A special test for convergence within the fixed-point precision being used (quit if $z_k = z_{k-1}$) accounted for 4314 points (29%). An overall cycle test (quit if $z_k = z_0$) accounted for only 4 points. The remaining 1422 points (9%) were cut off by the

dwell limit. An article about this program will appear in the next issue, as will an exchange between Mark Bolme and RS about the pros & cons of including the tricks in the dwell calculation.
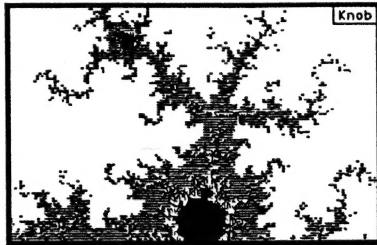
I am also working on a technique for enormously speeding up the process of generating an $\mathcal{M}$-image: I hope to reduce the time to produce an $\mathcal{M}$-image at the resolution limit for 70mm film (3000x5000 pixels) by a factor of 100 or more over the point-by-point method.

MOVIES & VIDEOS

Eskin Huff of Gaithersburg MD is "interested in making a video in real time in which one zooms in to an interesting area -- while the colors are cycling as can be accomplished in Deluxe Paint."

I am extremely interested in making a movie or video along similar lines, but perhaps differing in detail (e.g. how coloring is done). A group of us are attempting to get a grant or other funding for this project. The major problem will reducing the time to produce one frame to an acceptable value, without sacrificing resolution either in space, in time (dwell limit), or in precision (number of bits used in the arithmetic calculations). Approaches include the technique referred to above, and special hardware using e.g. DSP (digital signal processing) chips.

Bear in mind that as you attempt higher and higher magnification, you need more and more precision (at least enough to resolve the coordinate difference between adjacent pixels, and probably a good bit more), and higher and higher dwell limit (for frames centered on the boundary of $\mathcal{M}$).

CONTRIBUTIONS

Your contributions are welcome, including pictures, algorithms, programs, articles (technical or otherwise), references to articles or books on $\mathcal{M}$.

I can cover the elementary math relating to $\mathcal{M}$, but am a bit out of my depth when it comes to rational radii and differential geometry lingo (e.g. "the equivalence which is tangent to the identity at infinity"), etc. Any of you out there who can handle

same are cordially invited to contribute material on this level.

Personally, I'd like to see an elementary self-contained proof (or even sketch) of the connectivity of $\mathcal{M}$.

Douglas Geehan has offered to contribute to the newsletter in any way he can. Thanks, Douglas; I'll get back to you as things develop.

Eskin Huff uses a program for the Amiga taken from the "Fred Fish disks", and has a slide show in color. What are the Fred Fish disks, Eskin? If you send transparencies (positive or negative) of some color shots, maybe we can include some copies in a future issue.

Marty Skavlan suggests that we publish assembly language routines and implementation for various personal computers. I will publish my dwell routine in Assembler for the 68000 next time, Marty.

Thomas Bank suggests that I consider starting a Mandelbrot Computer Bulletin Board. I might, if someone can help me with the details: I have a computer, a modem, and a phone; but no program and no experience.

Michael West sent in a Mandelpic hand-colored from an ASCII printout -- an heroic job, indeed!

John Dewey Jones offers to contribute observations and speculations from the research into the set and related fractal topics undertaken in the Engine Research Department of the General Motors Research Laboratories. Would we like post-doctorate-level mathematical treatments? (I certainly would!) Tips on how to construct the set using only a slide-rule and sheet of graph paper? Proofs that the Set offers a means of communication with extra-terrestrial intelligence? Perhaps, John, someone will uncover deep within $\mathcal{M}$ a scene so improbable as to have theological significance (See **Contact**, by Carl Sagan -- toward the end of the book).
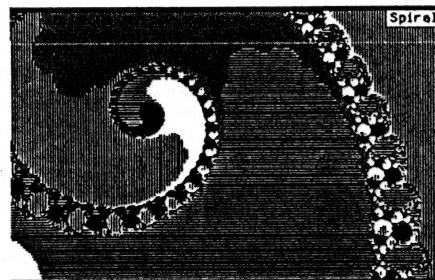
Melvin Saffran suggests reprinting relevant articles on $\mathcal{M}$. Such as, Melvin?

Mark Bolme enclosed literature describing **Fractal-Magic** and **MandelZoom**, and other products sold by his company, Sintar Software (PO Box 3746 / Bellevue WA 98009; (206) 455-4130).

Zoltan Szepesi has an Amiga, and an "excellent C-program by R.J. Mica". How fast is the program, Zoltan?

Pete Gwozdz has a "Mandelbrot Microscope" program for the IBM PC, and kindly sent me a copy.

E.T. Trigg sent a Mac diskette with a program to produce views of the Mandelbrot set on the Macintosh, and a slide show for viewing some pictures produced by it; four of these are displayed in this newsletter. He asks if there is a mathematical description of $\mathcal{M}$ that he could use to do gross checking for "inside-outside". If you mean checking whether a point is in $\mathcal{M}$ or not, ET, then other than checking whether its iterates get big, or loop, or stay small for the dwell limit, this gets into some interesting math. There are of course some special cases, e.g. all $|c| \leq .25$ are in, $|c| > 2$ are out.

BIBLIOGRAPHY

Computer Recreations, A.K. Dewdney, Scientific American, August 1985 (p16 ff and cover). Most of us "discovered" the Mandelbrot set by reading this column. For those of you that haven't read it, do, by all means. It's a short piece containing lots of useful information. The pictures, by Peitgen and Richter, are stunning.

The Fractal Geometry of Nature, Benoit B. Mandelbrot, W.H. Freeman & Co., New York (1983). ISBN 0-7167-1186-9. This is the basic work on fractals by the pioneer, Mandelbrot. The book is a feast for the eyes and the mind, and a "must" for anyone who wants to dig deeper into the world of fractals.

Deterministic Chaos, Heinz Georg Schuster, Physik-Verlag, 1984 (220 pages plus plates). This is a "heavy" book about physical systems which behave chaotically, i.e. with disorder and irregularity. There seems to be a deep connection between the dynamics of such systems and the process which underlies $\mathcal{M}$. Plates IX through XV are gorgeous renditions of $\mathcal{M}$-views, taken from Peitgen and Richter (1984). **Deterministic Chaos** is available in this country as ISBN # 0-89573-223-8 from VCH Publishers, 303 NW 12th Avenue, Deerfield Beach, FL 33441-1705.

Harmonie in Chaos und Kosmos, and **Morphologie komplexer Grenzen; Bilder aus der Theorie dynamischer Systeme**, Peitgen, H.O. and Richter, P.H. (1984). Many of you will recall that Peitgen and Richter did the beautiful $\mathcal{M}$-views that appeared on the cover of the August 1985 issue of Scientific American and in Dewdney's column therein. I haven't seen these catalogues, but Schuster says that they can be obtained from: Forschungsschwerpunkt: Dynamische Systeme, Universität Bremen, D-2800 Bremen, F.R.G. I have written for them, and will report on them in a later issue.

The Beauty of Fractals, H.-O. Peitgen and P.H. Richter. Springer-Verlag New York, Inc. (1985/110 pp/184 figs. in 221 sep. illus. mostly in color) ISBN 0-387-15851-0; $35.00 (up from $29.50).

"This book is an unusual attempt to publicize the field of Complex Dynamics, an exciting mathematical discipline of respectable tradition that recently sprang to new life under the impact of modern computer graphics. Where previous generations of scientists had to develop their own inner eye to perceive the abstract aesthetics of their work in lonely enjoyment of what seemed forever inaccessible to the uninitiated, the astounding pictures assembled here invite a general public to share in a new mathematical experience, to revel in the charm of fractal frontiers."

"In 88 color pictures, and many more black and white illustrations, the authors present variations of a theme whose repercussions reach far beyond the realms of mathematics. They show how structures of unseen complexity unfold by repeated action of simple rules, how territorial boundaries of bewildering entanglement may be generated by straightforward competition processes. The implied unpredictability of many details of these processes, in spite of their complete determination by the given rules, reflects a major challenge to prevailing scientific conceptions."

"Contributions by noted experts complement the book. B.B. Mandelbrot gives a very personal account of his discovery of the Mandelbrot set, and on the history of fractals in general. H.W. Franke, a pioneer in computer graphics, comments on the relationship between science and art. G. Eilenberger, as a theoretical physicist, points to the changes that our understanding of natural phenomena may undergo in the study of nonlinear processes.
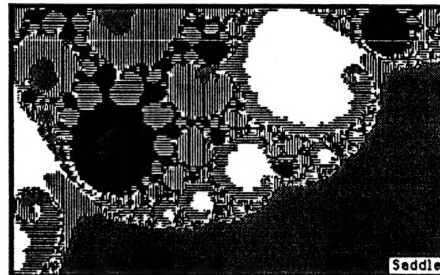
Finally A. Douady, one of the leading experts in the mathematics of the Mandelbrot set, recounts what is known about this amazingly complex object and what is not."

We believe that this book should be in the hands of everybody interested in fractals and $\mathcal{M}$. You can obtain it from your bookstore for $35.00; or from AMYGDALA / Box 219 / San Cristobal, NM 87564, for the reduced price of $31.50, postpaid.

SUBSCRIPTION

This zero-th issue is being sent out free. If you would like to subscribe for subsequent issues, send $15 for ten issues (U.S. and Canada; $25/10 issues elsewhere) to:
AMYGDALA
BOX 219
SAN CRISTOBAL, NM 87564
in the form of a check payable to "Amygdala".

PRODUCTS

ARTMATRIX (PO Box 880 / Ithaca, NY 14851) has Mandelpix and Fortran programs for generating them for sale. They have a catalog containing a few sample color shots. The resolution is impressive, but I found the color schemes a bit on the garish side, especially when compared to the beautifully sensitive renderings of Peitgen & Richter.

PLOTTING $\mathcal{M}$.

Dr. Henry Casson of Portland, Oregon has an IBM PC program to plot $\mathcal{M}$-views in relatively high resolution (769x1025 pixels) on an HP 7470A plotter. He sent a nice rendition of $\mathcal{M}$ done on the plotter in green and black; it is reproduced in gray scale on page 4. Thanks, Henry; further contributions will be welcome. Any ideas about how to get color copies into the newsletter inexpensively?

I also have a plotter (Western Graphtec MP-1000) which has a 360x270 mm. plotting area, with a 0.1 mm. step size. I am working on a system for plotting details of $\mathcal{M}$ in high resolution using this plotter: 2700x2000 pixels, using a Rapidograph- type pen with a 0.13 mm. tip.

Anyone else interested in plotters?

3) Dr. Henry Casson